

Uncertainty Propagation Quick Reference

Stony Brook Physics Labs

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Initial Uncertainty Formulas

Uncertainty in any particular measurements (based on repeated measurements):

$$\sigma_{x_i} = \sqrt{\frac{(x_1 - \bar{X})^2 + (x_2 - \bar{X})^2 + \dots + (x_N - \bar{X})^2}{N - 1}}$$

Uncertainty in the average of several measurements (based on those measurements):

$$\sigma_{\bar{X}} = \sqrt{\frac{(x_1 - \bar{X})^2 + (x_2 - \bar{X})^2 + \dots + (x_N - \bar{X})^2}{N(N - 1)}}$$

Basic Uncertainty Propagation Formulas

Operation	Absolute Error Formula	Relative Error Formula
Addition of constant	$\sigma(A + c) = \sigma_A$	(None)
Multiplication by constant	$\sigma(cA) = c \sigma_A$	$\sigma_{\text{rel}}(cA) = \sigma_{\text{rel}}(A)$
Addition & Subtraction	$\sigma(A \pm B) = \sqrt{\sigma_A^2 + \sigma_B^2}$	(None)
Multiplication	$\sigma(AB) = AB \sqrt{\left(\frac{\sigma_A}{A}\right)^2 + \left(\frac{\sigma_B}{B}\right)^2}$	$\sigma_{\text{rel}}(AB) = \sigma_{\text{rel}}\left(\frac{A}{B}\right)$
Division	$\sigma\left(\frac{A}{B}\right) = \left \frac{A}{B}\right \sqrt{\left(\frac{\sigma_A}{A}\right)^2 + \left(\frac{\sigma_B}{B}\right)^2}$	$= \sqrt{\sigma_{\text{rel}}(A)^2 + \sigma_{\text{rel}}(B)^2}$
Powers	$\sigma(A^n) = n A ^{n-1}\sigma(A)$	$\sigma_{\text{rel}}(A^n) = n \sigma_{\text{rel}}(A)$
Inversion	$\sigma\left(\frac{1}{A}\right) = \frac{\sigma_A}{A^2}$	$\sigma_{\text{rel}}\left(\frac{1}{A}\right) = \sigma_{\text{rel}}(A)$

Generalized Sum & Product Propagation

For your convenience, here are some more general rules:¹

$$\sigma(c_1A \pm c_2B \pm c_3C) = \sqrt{|c_1|^2\sigma_A^2 + |c_2|^2\sigma_B^2 + |c_3|^2\sigma_C^2}$$
$$\sigma_{\text{rel}}\left(\frac{cA^{n_1}B^{n_2}}{C^{n_3}}\right) = \sqrt{|n_1|^2\sigma_{\text{rel}}(A)^2 + |n_2|^2\sigma_{\text{rel}}(B)^2 + |n_3|^2\sigma_{\text{rel}}(C)^2}$$

¹To extend to more variables, just add more things under the square root. For the first formula, addition and subtraction always work the same. For the second, multiplication and division also always work the same, and multiplying by a constant never affects things (since we're dealing with relative error).