# A Fictional Measurement of the Acceleration due to Earth's Gravity

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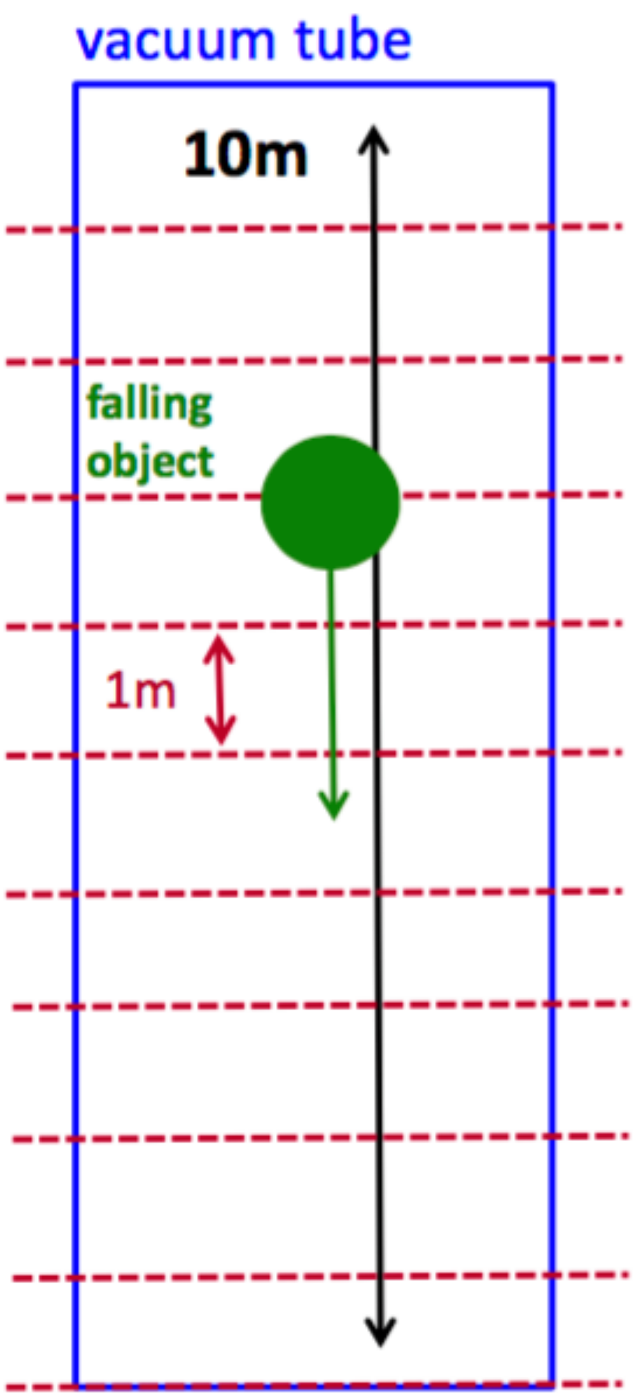
# Motivation of Experiment

- The goal of every experiment is to test a hypothesis
- In this experiment we want to measure the acceleration due to gravity (or our hypothesis for the law governing the change of velocity per time)

$$h(t) = \frac{1}{2}gt^2$$

- We therefore need to know the time it takes an object to travel a known distance under the influence of gravity
- Our experiment will consist of dropping an object from a specific height and recording the time from release until it hits the ground

# Setup of the Experiment




- You will drop a massive object in a 10 meter long vacuum tube (neglect air resistance)
- You precisely know the position of the markers (no uncertainty in the position)
- You will measure the time from release to when the object passes each successive meter mark
- You measure time with a stopwatch and therefore, this measurement has uncertainty
- Each time measurement has the same uncertainty

$$\sigma_t = 0.05\ s$$

# Recorded Data

| Distance [m] | Time [s] |
|---|---|
| 0 | 0.0 |
| 1 | 0.43 |
| 2 | 0.6 |
| 3 | 0.73 |
| 4 | 0.91 |
| 5 | 1.05 |
| 6 | 1.1 |
| 7 | 1.15 |
| 8 | 1.26 |
| 9 | 1.26 |
| 10 | 1.47 |

# Analysis

- We said before the goal of every experiment is to test a hypothesis
- Now that we collected data we want to see if it agrees with the hypothesis
- The first step is to see what is predicted by theory and then to analyze our results and compare to the data
- We will see that manipulating the equations to resemble a straight line is the best practice for analysis

# Finding the Fit Parameters

Clearly, we want to fit the data to a line with the form

$$y = A + Bx$$

So we need to solve for the A, B and their uncertainties and we do this by a method called “least-squares fitting”.

# Analysis

We are looking to solve the theory (the time) as a straight line

$$t(h) = \sqrt{\frac{2h}{g}}$$

To have as a straight line we take the log of both sides

$$\ln(t) = \frac{1}{2}\ln(h) + \frac{1}{2}\ln(\frac{2}{g})$$

Therefore, by comparison to

$$y = mx + b$$

We find the following for our parameters

$$y = \ln(t) \qquad m = \frac{1}{2} \qquad x = \ln(h) \qquad b = \frac{1}{2}\ln(\frac{2}{g})$$

# Finding the Intercept "A"

To calculate the intercept of the best fit line

$$A = \frac{1}{\Delta}\left(\sum \frac{x_i^2}{\sigma_i^2} \sum \frac{y_i}{\sigma_i^2} - \sum \frac{x_i}{\sigma_i^2} \sum \frac{x_i y_i}{\sigma_i^2}\right)$$

where

$$\Delta = \sum \frac{1}{\sigma_i^2} \sum \frac{x_i^2}{\sigma_i^2} - \left(\sum \frac{x_i}{\sigma_i^2}\right)^2$$

where each sum is obviously treated as

$$\sum \frac{x_i^2}{\sigma_i^2} = \frac{x_1^2}{\sigma_1^2} + \frac{x_2^2}{\sigma_2^2} + \frac{x_3^2}{\sigma_3^2} \ldots \frac{x_i^2}{\sigma_i^2}$$

# Finding the Slope "B"

To calculate the slope of the best fit line

$$B = \frac{1}{\Delta}\left(\sum \frac{1}{\sigma_i^2} \sum \frac{x_i y_i}{\sigma_i^2} - \sum \frac{x_i}{\sigma_i^2} \sum \frac{y_i}{\sigma_i^2}\right)$$

where again

$$\Delta = \sum \frac{1}{\sigma_i^2} \sum \frac{x_i^2}{\sigma_i^2} - \left(\sum \frac{x_i}{\sigma_i^2}\right)^2$$

The uncertainties in these values are also calculated from known least-squares equations.

# Uncertainties in Fit Parameters

The uncertainty in the intercept of the best fit line

$$\sigma_A^2 = \frac{1}{\Delta}\sum(\frac{x_i^2}{\sigma_i^2})$$

The uncertainty in the slope of the best fit line

$$\sigma_B^2 = \frac{1}{\Delta}\sum(\frac{1}{\sigma_i^2})$$

# Chi-Squared

For a linear fit, the chi-squared value is given by

$$\chi^2 = \sum(\frac{1}{\sigma_i}(y_i - a - bx_i))^2$$

I will spare you the theory but this value is essentially a the “goodness-of-fit” parameter. It tells us how likely that our observed data points come from the parent distribution (a model), which is a line in this case.

A “good fit” is usually one where the reduced chi-squared value is less than or equal to one