## Series and Parallel Circuits

The purpose of this lab is to study the potential difference and resistance across various points in series, parallel and combination circuits. You will verify the validity of the expressions for total resistance in each circuit.

## Equipment

- Digital multimeter
- Circuit board



## INTRODUCTION:

Components in an electrical circuit are in series when they are connected one after another, so the same current flows through them. Components are in parallel when they are in alternate branches of a circuit. When the resitors are connected parallel the voltage drop on all of the resistors is the same. Series and parallel circuits function very differently. For example, when using some types of older decorative holiday light circuits, if one lamp is removed, the whole string of lamps goes off. These lamps are in series. When a light bulb is removed in your home, the other lights stay on, because household wiring is normally in parallel.

In this lab we will be using a simple digital multimeter to measure voltage and resistance. The meter has a red and a black probe. These probes have to be connected to the meter as shown on the photos here. We will use two settings. The "DCV" setting (on the right) is to measure voltage and the " $\Omega$ " setting (on the left) is for measuring resistance.

The goal of this experiment is to study circuits made up of two resistors in series or parallel. You can then use Ohm's law to determine the
 equivalent resistance of the two resistors.

In this part, you will connect and investigate two resistors in series. You will verify if the potential difference (voltage) on the two resistors add up.

## PROCEDURE:

1. Wire the two resistors, $R_{1}$ and $R_{2}$, in series so that you obtain the circuit shown in Fig. 1. Do not connect the voltmeter yet. Note: The numbers in Figure 1 refer to the numbered terminals on the circuit board.
2. Set Switch 1, SW1, located below the battery holder on the circuit board to 3 V .
3. Have the instructor check the arrangements of the wires before proceeding.
4. Switch the digital multimeter to the "DCV" settings. In this mode it measures the voltage between the terminals.
5. Press the probes of the voltmeter to terminals 2 and 6 (as shown in the Figure.)


Figure 1
6. Ask your lab partner to close the circuit by pressing the switch (in 22-23 terminals). Record the voltage in Table 1. We will call this $\mathrm{V}_{26}$.
7. Repeat the procedure using terminals 2 and 3 , and record $V_{23}$.
8. Repeat the procedure using terminals 7 and 6 , and record $V_{76}$.
9. If any of the voltages is negative, check the position of the voltage probes. The red probe should be always closer to the positive terminal of the battery than the black probe. ("Close" in the sense of following the current flow in the circuit.)
10. Estimate an error for the values of the measured voltage and current using any fluctuations of these values in the multimeter. If there are no fluctuations, the error is $\pm 1$ of the last digit. For example, if the voltage is 12.27 mV , the error is $\pm 0.01 \mathrm{mV}$.

TABLE 1

| Terminals $\mathbf{2}$ and 6 |  | Terminals 2 and 3 |  |  | Voltage 7-6 [V] |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{V}_{26}[\mathrm{~V}]$ | $\Delta \mathrm{V}_{26}[\mathrm{~V}]$ | $\mathrm{V}_{23}[\mathrm{~V}]$ | $\Delta \mathrm{V}_{23}[\mathrm{~V}]$ | $\mathrm{V}_{76}[\mathrm{~V}]$ | $\Delta \mathrm{V}_{76}[\mathrm{~V}]$ |  |
|  |  |  |  |  |  |  |

Answer these questions:

1. Does the $V_{26}=V_{23}+V_{76}$ relationship hold?
2. We call the resistance between terminals 2 and $6 R_{\text {total. }}$. If the $V_{26}=V_{23}+V_{76}$ relationship indeed holds, and the current flowing through resistor $R_{1}$ is the same as the current flowing in resistor $\mathbf{R}_{\mathbf{2}}$, what is the relationship between $\boldsymbol{R}_{1}, \boldsymbol{R}_{\mathbf{2}}$ and $\boldsymbol{R}_{\text {total }}$ ?

This time we will make a really simple circuit. We remove every other wires, except for the one connecting terminals 3 and 7 , see Fig. 2.


## PROCEDURE:

Figure 2

1. Switch the digital multimeter to the " $\Omega$ " settings. In this mode it measures the resistance between the terminals.
2. Measure the value of $R_{1}$ by pressing the probes of the multimeter to terminals 2 and 3 .
3. Measure the value of $R_{2}$ by pressing the probes of the multimeter to terminals 6 and 7 .
4. Measure the value of the series resitance $R$ by pressing the probes of the multimeter to terminals 2 and 6 (see Figure).
5. If your readout of the resistances did not fluctuate (they should not!), the error is $\pm 1$ of the last digit.
6. Record the values (together with units) in the first row of Table 2
7. Connect the resistors between terminals 14,15 and 16,17 in series. To avoid confusion with the previous data, call the resistance between 14 and $15 R_{1}{ }^{\prime}$, the resistance between 16 and $17 R_{2}{ }^{\prime}$ and the total resistance $R^{\prime}$
8. Repeat the same procedure to obtain $R_{1}{ }^{\prime}, R_{2}{ }^{\prime}$ and $R^{\prime}$ and record them in the second row of the Table.

Technical note: When you measure resistance with the simple method we use here, the measurement also includes the resistance of the lead wires and the contacts. Ideally, these resistances should be much lower than the measured resistance. When the measured resistor is in the $10 \Omega$ range the contact resistance (how hard you press the probe to the terminal) may come into play. There is no such problem when the resitors are in the $10^{4} \Omega$ range (like $R_{1}{ }^{\prime}, R_{2}{ }^{\prime}$ ).

TABLE 2

| Terminals 2 and 6 <br> Terminals 14 and 17 |  | Terminals 2 and 3 <br> Terminals 14 and 15 |  | Terminals 7 and 6 <br> Terminals 16 and 17 |  | Calculated Series <br> Resistance |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{R}, \boldsymbol{R}^{\prime}$ | $\Delta \boldsymbol{R}, \Delta \boldsymbol{R}^{\prime}$ | $\boldsymbol{R}_{1,}, \boldsymbol{R}_{1}^{\prime}$ | $\Delta \boldsymbol{R}_{1}, \Delta \boldsymbol{R}_{1}^{\prime}$ | $\boldsymbol{R}_{\mathbf{2}}, \boldsymbol{R}_{12}{ }^{\prime}$ | $\Delta \boldsymbol{R}_{2}, \Delta \boldsymbol{R}_{\mathbf{2}}{ }^{\prime}$ | $\boldsymbol{R}_{\text {calc }}$ <br> $\boldsymbol{R}_{\text {calc }}^{\prime}$ | $\Delta \boldsymbol{R}_{\text {calc }}$ <br> $\Delta \boldsymbol{R}_{\text {calc }}^{\prime}$ |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |

Use this equation to calculate the total theoretical resistance you should obtain when you connect your two resistors in series and compare it with the total experimental resistance.

$$
\begin{equation*}
R_{\text {calc }}=R_{1}+R_{2} \tag{1}
\end{equation*}
$$

To calculate the error of $R_{\text {total }}$, use Eq. (6) of the "Uncertainty, Error and Graphs" lab manual, http://skipper.physics.sunysb.edu/~physlab/doku.php?id=phy133:error and uncertainty:

$$
\Delta R_{\text {calc }}=\sqrt{\Delta R_{1}^{2}+\Delta R_{2}^{2}}
$$

## Answer these questions:

1. Is the measured resistance the same as the value written next to it? If not, what can be an explanation? (Hint: these resistors are really cheap.)
2. Is the experimental equivalent resistance $R$ consistent with the calculated value $R_{\text {total }}$ ? (i.e. Are the two values equal within the experimental error?)
3. When do you get better agreement with the theoretical value: If you use the measured resistances or if you use the values written next to the resistors?

PART 4.- Resistors in parallel
In this part, you have two resistors in parallel. We will investigate the parallel circuit shown in Fig. 3. In particular, we will see if the resistance of two in parallel resistors $R_{1}$ and $R_{2}$ satisfies
$\frac{1}{R_{\text {calc }}}=\frac{1}{R_{1}}+\frac{1}{R_{2}}$


Figure 3

Please note that we have several different ways of setting up parallel circuits, and they are all equivalent, as long as the resistance of the lead wires is very much smaller then the resistance of the resistor we are dealing with. Under this condition, the electric potential difference between the terminals connected by wires is zero, and it does not matter how the connection was made. The photos below show two different ways of realizing the circuit shown in Figure 3 and there are many more ways to do this. The cicuit in the right and left will have the same resistance between terminals 1 and 22.


## PROCEDURE:

1. Wire the two resistors, $\mathrm{R}_{1}$ and $\mathrm{R}_{2}$, in parallel so that you obtain the circuit shown in Fig. 3. Note: The numbers in the Figure refer to the numbered terminals on the circuit board.
2. Switch the digital multimeter to the " $\Omega$ " settings.
3. Using the multimeter, measure the resistance between terminals 1 and 22 . We call this resistance $R$.
4. Remove the connecting wires and measure the values of $R_{1}$ and $R_{2}$ separately.
5. Record the values in Table 3.
6. If your readout of the resistances did not fluctuate (they should not!), the error is $\pm 1$ of the last digit. Record the error values in Table 3.

The rest of Table 2 is filled out with calculated values. First, according to Eq. (2) we need to calculate the inverse of the resitances $S_{1}=1 / R_{1}$ and $S_{2}=1 / R_{2}$. We use Eq. (7) of the "Uncertainty, Error and Graphs" manual, and we conclude that the relative error of $R$ and the relative error of $S$ is the same. (This follows from Eq (7) if we take A=1 with no error.) Therefore $\Delta R_{1} / R_{1}=\Delta S_{1} / S_{1}$ and $\Delta S_{1}=S_{1}\left(\Delta R_{1} / R_{1}\right)=\Delta R_{1} / R_{1}{ }^{2}$. We have a similar expression for $\Delta S_{2}$.

TABLE 3

| $\mathrm{R}_{1}$ | $\Delta \mathrm{R}_{1}$ | $\mathrm{~S}_{1}=1 / \mathrm{R}_{1}$ | $\Delta \mathrm{~S}_{1}$ | $\mathrm{R}_{2}$ | $\Delta \mathrm{R}_{2}$ | $\mathrm{~S}_{2}=1 / \mathrm{R}_{2}$ | $\Delta \mathrm{~S}_{2}$ | R | $\Delta \mathrm{R}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |  |

Table 4 is filled out with calculated values. According to Eq. (2) $S_{\text {calc }}=\frac{1}{R_{\text {calc }}}=\frac{1}{R_{1}}+\frac{1}{R_{2}}$. The error of this quantity is $\Delta S_{\text {calc }}=\sqrt{\Delta S_{1}^{2}+\Delta S_{2}^{2}}$. Finally, we calculate $R_{\text {calc }}=1 / S_{\text {calc }}$ and $\Delta R_{\text {calc }}=\Delta S_{\text {calc }} / S_{\text {calc }}{ }^{2}$.

TABLE 4

| $S_{\text {calc }}$ | $\Delta S_{\text {calc }}$ | $R_{\text {calc }}=1 / S_{\text {calc }}$ | $\Delta R_{\text {calc }}$ |
| :--- | :--- | :--- | :--- |
|  |  |  |  |

Is the total experimental resistance $R$ consistent with the calculated value $R_{\text {calc }}$ ? To answer this question indicate $R \pm \Delta R$ and $R_{\text {calc }} \pm \Delta R_{\text {calc }}$ on this ruler (in units of $\mathbf{k} \Omega$ ).


PART 6.- Resistors in series and parallel (combination)
In this part, you will connect several resistors in both series and parallel. You should be able to find the total calculated resistance by identifying which resistors are in series and which are in parallel. At the end, in your notebook, you will reduce the circuit with only one resistor and find this value by using equations (1) and (2).

## PROCEDURE:

1. Wire resistors $R_{1}=10 \Omega, R_{2}=10 \Omega, R_{3}=51 \Omega, R_{4}=51 \Omega$, and $R_{5}=68 \Omega$ as is shown in the following circuit.
2. Switch the digital multimeter to the " $\Omega$ " settings.
3. Using the multimeter, measure the resistance between terminals 1 and 9 . We call this resistance $R$.
4. Remove the connecting wires and measure the value of each resistor separately by pressing the probes of the multimeter to the terminals.
5. Calculate $R_{\text {calc }}$ using Equation (1) when the resistors are in series and (2) when they are in parallel. You will need to identify which resistors are in series and which are in parallel. Draw equivalent circuits until you get the final circuit with only one resistor.
6. Do not calculate errors for this part.


Figure 4

Answer these questions:

1. Is the total experimental resistance $R$ consistent with the calculated value $R_{\text {call }}$ ?
2. Why was it important to remove the connecting wires before measuring the individual resistances?
