Introduction to Measurement and Error Analysis

(PHY 115 and 117)

Introduction

In the sciences, measurement plays an important role. The accuracy of the measurement, as well as the units, help scientists to better understand phenomena occurring in nature. The better time is measured, for example, the better we can calculate the vertical of a basketball player or the speed of an F1 racecar.

In this lab, we will use different instruments to measure the volume and mass of metal cylinders and calculate the error for each measurement, paying attention to significant figures as well. From these measurements, we will determine the density of each metal and compare them to the known values.

Equipment List

At least 3 cylinders of the same metal, 1 ruler, 1 set of vernier calipers, 1 micrometer, 1 graduated cylinder

Procedure

1. There will be at least three cylinders of the same metal at your lab station. Both you and your lab partner will independently use the ruler to measure the diameter (d) and length (L) of each cylinder. Record the minimum uncertainty for each measurement.

2. Repeat these measurements with a vernier caliper and, if possible, a micrometer. Record the minimum uncertainty. Again make sure that the measurements are independent of each other. The TA will show you how to use these instruments.

3. Make an average of the data for each instrument used.

4. Calculate the uncertainty of each of these measurements by taking the difference between you and your partner's measurements and dividing them by two. Take the greater value between this calculation and the minimum uncertainty you recorded for your error of this measurement.

5. Measure the volume directly by immersing each metal cylinder into the graduated cylinder. Record both the initial and final volumes on the graduated cylinder as well as the uncertainty.

6. Determine the volume of each cylinder using the fact that

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and the averages from each instrument. Calculate the error for each volume as well.

7. Measure the mass m of each cylinder using a pan balance and record the uncertainty.

8. Calculate the density ρ of each cylinder for each instrument used to measure the volume. Calculate the error for the density using the information in the Error and Uncertainty section of the lab manual. Density is mass divided by volume.

9. For each cylinder, take an average of the volumes that you calculated. Obtain the error for this average by taking the difference of the largest and smallest volume (without error included) and divide by two.

10. Plot mass vs. volume for each of the cylinders with error bars. Determine the best fit slope as given by the following Error and Uncertainty section with error. Compare this value with the values you calculated for density as well as the density given by the TA.

Questions

Are the values you calculated for the density within experimental uncertainty of the given value?

Were your values for density precise? Accurate? Both? Why or why not?

What assumptions have been made about the cylinders?

Physics 115 Error and Uncertainty

In physics, as well as other sciences, every measurement comes with some sort of uncertainty. In reporting the results of an experiment, it is necessary to report this uncertainty as well as the measured experimental value. This is just as true when measuring how fast Michael Johnson runs the 200 meters as measuring the radiation background in the universe. For this course, we are going to use a simple approach to estimate these uncertainties and calculate the overall error of an experiment.

One common misconception is that the experimental error is the difference between our measurement and the accepted (or “official”) value. What is meant by error is the estimate of the *range* of values within which the *true* value is likely to lie. The range is determined by the instruments used in the experiment and the methods we use.

Error

If we denote a quantity that is measured through an experiment as X, then the error is called ΔX (read as: delta X). If X represents the time swam in the 100 meter freestyle, we might say the time is $t = 25.1$ 0.1 s, where the central value for time is 25.1 s and the error $\Delta t = 0.1$ s. Both the central value and the error must be reported in your lab report. Note that in this example, the central value is given to three significant figures. Do not give figures beyond the first digit in your uncertainty for any quantity. They do not have any meaning and are therefore misleading.

In the example given in the previous paragraph, the error quoted for the time is called the *absolute* error. There is also the *relative* error, which is defined as the ratio of the error to the central value. The relative error for the 100 meter time is $\Delta t / t = 0.1 / 25.1 = 0.004$. Notice that the relative error has no dimensions and should have as many significant figures as those that are known for the absolute error.

Random Error

Random error occurs because of small variations in the measurement process. Measuring the time of a pendulum’s period with a stopwatch will give different times because of small differences in your reaction time of starting and stopping the stopwatch at the beginning and end of a period. If this is random error, then the average of the measurements will get closer to the correct value as the number of trials increases. The correct result would be the average for our central value and we will take the error to be the standard deviation of the measurements, in the following approximation. In this lab, we will rarely take many measurements of a quantity. Instead, a few measurements are taken (3-5). To get the central value, we would take the average of these measurements. To get the error, the difference is taken between the largest and smallest values and then divided by two. Remember to go only to the first significant figure for the error and only to that decimal place for the central value.

Example: A football player is training for the combine, a display of talents that will help teams determine if they want to draft him. In a week of training, he has five times for the 40: 4.3s, 4.2s, 4.4s, 4.5s, 4.2s. The average over the week is 4.32s. The error Δt is calculated by (4.5s - 4.2s) / 2 = .15s. But because of the fact that we allow error to have one significant figure, $\Delta t = 0.2$s. Thus, the average time with error is $t = 4.3\text{s} \pm 0.2\text{s}$.

Systematic Error

Some sources of error are not random. For example, if you used a meter stick to measure the length of a friend's arm, the meter stick may be warped, or may have stretched, and you would never get a good value with that instrument. More subtly, the length of the meter stick might depend on temperature, and this measurement will be good only for the temperature that it was calibrated. In this course, systematic errors should be considered, but the effects in each lab are assumed to be small. However, if the value of a quantity seems to be far off from what you would expect, you should think about the possible sources of systematic error more carefully.

Propagation of errors

Often in the lab it is needed to combine two or more measured quantities, each of which has an error, to get a desired quantity. For example, you want to know the perimeter of a football field and you measured the length l and the width w with a tape measure. Since you know that the perimeter $p = 2(l + w)$. To get the error on p, you would need to know the errors estimated for l and w, Δl and Δw.

There are some rules for calculating errors of such combined, or derived, quantities. Suppose you have made primary measurements of A and B, and would like to get the best value and error for some derived quantity S. For addition and subtraction of the given quantities:

If $S = A + B$, then $\Delta S = \Delta A + \Delta B$.

If $S = A - B$, then $\Delta S = \Delta A + \Delta B$ as well.

A more refined rule for either case $S = A \pm B$ is

$$\Delta S = \sqrt{(\Delta A)^2 + (\Delta B)^2}$$

which takes into account the fact that some random error will cancel out.

For multiplication or division of measured quantities:

If $S = A * B$, then $\Delta S / S = \Delta A / A + \Delta B / B$.

If $S = A / B$, then $\Delta S / S = \Delta A / A + \Delta B / B$ as well.

Note that these two take into account the *relative* errors of A and B. Again, if the errors in A and B partly cancel out, you can use

$$\frac{\Delta S}{S} = \sqrt{\left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2}$$

Now an example needs to be done. For the equation for the perimeter above, $p = 2(l + w)$, the error is $\Delta p = 2\Delta l + 2\Delta w$. This is because 2 is a constant, so what you have is the case where $S = A + B$. Suppose you were interested also on the area of such a rectangle. Then, the error for the area (*Area = l * w)* would be:

$$\Delta\, Area = Area\left(\frac{\Delta l}{l} + \frac{\Delta w}{w}\right)$$

Obtaining Values from a Graph

In some of these labs, you will be asked to graph data to obtain results from the slope of the graph. For an example of this, we will use the example of a drag racer. The data that we would normally take would be the distance *l* that the drag racer has gone in a time t. From this data, *l* would be plotted on the y axis versus t^2 on the x axis (see figure below). The slope of this line would allow us to determine the acceleration that the drag racer experiences, $l = (a * t^2) / 2$. In constructing the graph, plot a point at the central (x, y) values for each of your measurements. Small horizontal bars should be drawn with a length that is equal to the absolute error in the x direction (t^2) in both, the positive and negative directions. Similarly, small vertical bars should be drawn with a length that is equal to the absolute error in the y direction (*l*) in both, the positive and negative directions.

Because of the error bars, we wish to find the slope that is most likely to fit the data. To do this, we draw two slopes. The first slope is drawn to pass through or near the upper error bars. The second is drawn to pass through or near the lower error bars. The average of these two slopes gives the best fit acceleration. Half the difference of these two slopes gives the absolute error of the acceleration.